## Row Echelon Form (REF)

A matrix is in row echelon form if and only if the first (leftmost) non-zero entry in each row is 1 (called the leading 1), the leading 1 in each row (except row 1) is to the right of the leading 1 in the row above it, and all rows which contain only 0 are below all rows which contain any non-zero entry.

Γ	1	3	0	-2	4]	[	- 1	3	0	-2	4	Γ	1	3	0	-2	4]	[ 1	3	0	-2	4
	0	1	7	4	0		0	1	7	4	0		0	1	7	0	0	0	1	7	4	0
	0	0	-1	5	6		0	1	4	-3	-2		0	0	1	0	-2	0	0	0	1	-2
	0	0	0	1	3		0	0	1	1	3		0	0	0	1	3	0	0	0	0	0

Reduced Row Echelon Form (RREF)

A matrix is in reduced row echelon form if and only if

it is in row echelon form,

and all columns which contain a leading 1 contain only 0 in all other entries.

1	0	-1	-2	4]	[ 1	0	0	0	4]	[ 1	0	-3	0	4]
0	1	0	4	0	0	1	0	0	0	0	1	8	0	0
0	0	1	5	6	0	0	1	0	-2	0	0	0	1	6
0	0	0	0	0	0	0	0	1	3	0	0	0	0	0

Gaussian Elimination

- Step 1: Find the first (leftmost) column which contains a non-zero entry
- Step 2: Choose a pivot in that column (to be used to replace all lower entries in that column with 0)
- Step 3: SWAP to move the pivot's row to the top
- Step 4: SCALE to turn the pivot into 1
- Step 5: REPLACE each row below the pivot's row
  - by adding the multiple of the pivot's row which gives a 0 under the pivot
- Step 6: Cover up the pivot's row & repeat the entire process (stop when matrix is in row echelon form)

Gauss-Jordan Elimination (after matrix is in row echelon form)

- Step 7: Find the last (rightmost) column which contains a pivot (leading 1)
- Step 8: REPLACE each row above the pivot's row

by adding the multiple of the pivot's row which gives a 0 above the pivot

Step 9: Cover up the pivot's row & repeat the entire process (stop when matrix is in reduced row echelon form)

## The following examples should not require fractions if solved using the processes above.

Example 1:	Example 2:	Example 3:
3x + 2y - z = -1 5x + y - 3z = -2 2x + 4y + 2z = 2	4x + 6y - 3z = -153x + 4y + z = 11-x - 2y + z = 1	3x + 4y - 11z = -17 2x + y - 4z = 5 -x - 2y + 5z = -9
Example 4:	Example 5:	
3x + 5y - 9z = 14 2x - 3y + 13z = 3 -x + 2y - 8z = -1	2x + 4y + 11z = 10 x + 2y + 7z = 5 3x + 4y + 9z = 13	